Financial Markets Microstructure
Final Re-Exam with Solutions

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Problem 1

The Parlour model explores traders' choices between market and limit orders. Its bottom line, as we argued in class, is that limit orders result in more favorable prices but at the cost of the execution risk. In particular, we ignored the potential effects of asymmetric information. Suppose now instead that with some probability public information (news) can arrive between periods t and $t+1$ (but not at any other time, and all of these facts are commonly known). This will produce an asymmetry in the sense that trader at $t+1$ will be able to act upon this information, while limit orders submitted by trader t are independent of this news.

Answer the following questions using convincing intuitive arguments. Proceed via backwards induction: i.e., holding previous traders' strategies fixed, state how the strategy of a trader in a given period changes. You do not need to analyze a formal model, but you are welcome to do so if you want to.

1. How will the behavior of traders who arrive at $t+2$ or later change due to the possible arrival of news?

2. What about the trader who arrives at $t+1$?

3. What about the trader who arrives at t?

4. Now suppose that if the period-t trader submitted a limit order, he can revise/cancel his limit order after the news arrives. How do your answers to parts 1-3 change?

Solution:

1. All quotes will reflect the new information, but the trade-off between market and limit orders will remain the same.

2. The trader at $t+1$ will have a wider range of possible asset valuations, since in addition to idiosyncratic valuation he will know the innovation to the fundamental value. Hence holding fixed the limit order strategy of t-trader, the trader at $t+1$ will use market orders with higher probability. This effect is state-dependent: good news increases the probability of a market buy order and decreases the probability of any sell order (if the distribution of idiosyncratic valuations y_t is not too strange). The effect is flipped for bad news.

3. Public information affects the strategic environment of time-t trader in two dimensions. On the one hand, his limit order, if submitted, has a higher execution probability. This makes limit orders more appealing and would lead to time-t trader using more limit orders and less market orders. On the other hand, limit orders can now be picked off: a sell limit order becomes more likely to be executed in case of "good news" about asset valuation – i.e., the t-trader sells too cheaply on average. This will make limit orders less appealing, since price improvement they yield may be completely offset by adverse selection.

   The formal analysis can help answer which of the two effects above dominates and whether time-t trader will end up using limit orders more or less actively relative to market orders, although the exact conclusion will depend on the assumptions about the distributions of y_t (the idiosyncratic component) and ϵ_t (the public news component).

4. If time-t trader can cancel and revise his limit order before it can be picked off by the $t+1$-trader, then the revision will incorporate the public information, and the expected payoff of time-t trader from submitting a limit order will remain exactly the same as in the absence of news. Therefore, his choice between market and limit orders will not be affected.

   If there is some chance that $t+1$-trader can manage to trade before t-trader can revise his quotes, then the effect from part 3 applies to the extent proportional to that chance. (Note parallels to the Budish-Cramton-Shim model.)

Problem 2

This question investigates the inventory risk in uncertain environments within the Stoll model framework. Consider a three-period model, $t \in \{0, 1, 2\}$. There is one asset, whose fundamental value evolves as $\mu_{t+1} = \mu_t + \epsilon_t$, where $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$. The respective μ_t is observed at the beginning of period t.

In periods $t \in \{0, 1\}$ the representative dealer must provide quote schedule $p(q)$ for any incoming order size q (where $q > 0$ means a buy order and $q < 0$ means a sell order). In period $t = 0$ one trader arrives for sure and submits an order denoted by q_0. In period 1 one trader arrives with probability λ and, if he does, submits an order $q_1 = -q_0$. In period $t = 2$ the asset is paid out: every owner of the asset receives a payment μ_2 per unit and the asset has no future value.

Dealer has mean-variance preferences over his final wealth w_2. I.e., in every period he maximizes

$$U(w_2) = \mathbb{E}[w_2] - \frac{\rho}{2}\mathbb{V}(w_2).$$

His initial position in the asset is neutral: $z_0 = 0$. The initial cash holdings c_0 are also normalized to zero. (The dealer can borrow cash and short the asset at no cost). The dealer behaves competitively (is a price-taker).

1. Consider period $t = 1$. Denote the dealer's position at the beginning of the period as z_1. Derive the dealer's quote schedule $p_1(q)$ given z_1.

2. What is the price at which trade will happen at $t = 1$?

3. Consider period $t = 0$. Derive the dealer's quote schedule $p_0(q)$.

4. Explain how $p_0(q)$ depends on λ and why.

   NOTE: if you could not solve parts 1-3, you can still try to make an educated guess here.

5. The problem assumes $q_1 = -q_0$, i.e., that the order flow is perfectly negatively autocorrelated. How justified is this assumption? How does it relate to the dealer's pricing decisions?

   NOTE: if you could not solve parts 1-4, you can still answer this question.

Solution:

1. This part is identical to the Stoll model we had in class. The dealer is a price-taker, so he chooses the optimal asset supply y_1 given any price p. We derive the resulting asset supply schedule $y_1(p)$, invert it and invoke market clearing condition $y_1 = q_1$ to derive the price schedule $p_1(q)$. Asset supply y_1 is chosen to maximize

$$\begin{aligned} U_1(y_1) &= \mathbb{E}[w_2] - \frac{\rho}{2}\mathbb{V}(w_2) \\ &= py_1 + \mu_1(z_1 - y_1) - \frac{\rho}{2}(z_1 - y_1)^2\sigma_\epsilon^2 \end{aligned}$$

   Taking the first order condition and substituting $q = y_1$, we end up with

$$p_1(q) = \mu_1 + \rho\sigma_\epsilon^2(q - z_1).$$

2. The problem states that $q_1 = -q_0$, meaning that $q - z_1 = 0$, so $p_1 = \mu_1$.

3. Whenever the dealer trades y_0 at $t = 0$, two continuations are possible: with probability λ an order $y_1 = -y_0$ will arrive at $t = 1$, in which case

$$w_2 = p_0 y_0 + p_1 y_1 = (p_0 - \mu_1) y_0.$$

With probability $1 - \lambda$ no trader will arrive at $t = 1$, so

$$w_2 = p_0 y_0 + \mu_2 z_2 = (p_0 - \mu_2) y_0.$$

Therefore, the dealer's objective function at time $t = 0$ is

$$U_0(y_0) = (p_0 - \mu_0) y_0 - \frac{\rho}{2} \left[\lambda \sigma_\epsilon^2 + (1 - \lambda) 2 \sigma_\epsilon^2\right] y_0^2 \tag{1}$$

Maximizing this expression w.r.t. y_0 for given p_0 and invoking the market clearing condition $q_0 = y_0$, we obtain

$$p_0(q) = \mu_0 + \rho(2 - \lambda)\sigma_\epsilon^2 q \tag{2}$$

4. Larger λ decreases the price impact coefficient. Higher λ means that the dealer faces a higher chance of unwinding his inventory at $t = 1$ and not staying stuck with it until $t = 2$. This reduces the dealer's exposure to volatility in the asset's fundamental value μ_t (which is priced because the dealer is risk-averse), and so reduces the dealer's required risk premium.

5. In the real world, order flow from traders is typically positively autocorrelated (although this depends on the time scale). This may be due to traders splitting their large orders and feeding them to the market as a series of small orders, or due to different traders acting upon the same piece of news. In this respect the assumption is disconnected from the real world.

   However, in this problem the dealer dislikes holding on to any inventory due to the risk this exposes him to, and this feeds into his pricing decisions so as to create negative autocorrelation in the order flow. E.g., if the dealer has purchased a unit of the asset at $t = 0$, then he would set lower prices $p_1(q)$ in the following period so as to incentivize buyers and discourage sellers, which creates respective pressure on the order flow.

   That said, we are looking at the representative dealer in this model – a fictional agent meant to represent the aggregate of all individual dealers in the market. While any single dealer can be non-trivially risk-averse, the market as a whole can absorb quite significant positions before distorting the prices due to inventory concerns. Therefore, it may be reasonable to believe that the representative dealer's degree of risk aversion is quite small, so this driver of negative autocorrelation should be weak.

Problem 3

Many real-world trading platforms have "circuit breakers" – automatic safeguards which halt all trading in an asset if its price changes too drastically over a short time. Using the material of the course, discuss what consequences can the existence of such circuit breakers have in terms of market outcomes (liquidity, traders' risk exposure and willingness to trade, price discovery).

Solution: The most basic arguments that can be made are as follows.

- Circuit breakers halt trade on a platform, which by definition kills all liquidity for the duration of the halt, thus existence of CBs can be seen as a liquidity risk.
- CBs are triggered by the excess volatility of the asset price, hence they effectively put an upper bound on this volatility (on the particular time frame of minutes, but not on the micro-scale of milliseconds, and not on the long-run volatility). This provides a form of insurance to traders who hold positions in this asset, and so decreases their risk exposure. This is likely to increase the investors' willingness to trade, more so for those who are not capable or willing to monitor the asset price continuously – i.e., the less sophisticated and the less informed investors. As we know, presence of such investors makes the market more liquid, and this boost to liquidity can reasonably outweigh the liquidity risk mentioned above, improving the overall market liquidity.
- By design, CBs may hurt price discovery, since the asset price can no longer adjust rapidly in response to news. Higher share of uninformed investors in the market mentioned above also slows down price discovery in the presence of CBs. Higher-order arguments can be made (e.g., "uninformed investors slow the price discovery down by just enough to avoid triggering CBs, which in the end improves the average speed of price discovery").

Many of the more subtle arguments, as well as empirical estimates of the actual effects, can be found in ESMA working paper No. 1, 2020 ("Market impacts of circuit breakers – Evidence from EU trading venues", available at `https://www.esma.europa.eu/sites/default/files/library/esmawp-2020-1_market_impacts_of_circuit_breakers.pdf`).